The Effect of neutrino on the Meta-Stability of Electroweak vacuum

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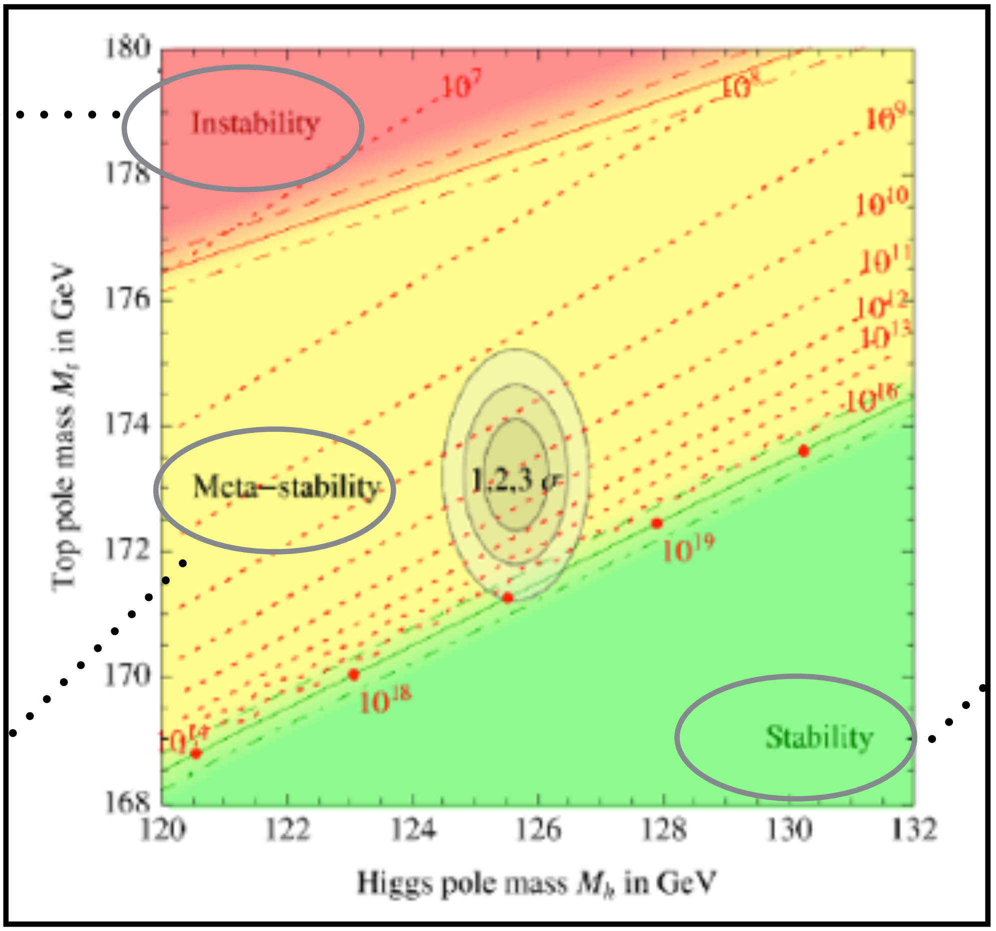
Introduction

EW Vacuum Stability in SM

Within SM, although electroweak(EW) vacuum is not absolutely stable,
since the lifetime is longer than the age of universe
the stability of EW vacuum is guaranteed.

EW vacuum
isn’t the global minimum
and
the lifetime is shorter than
the age of universe



Dario Buttazzo et.al[JHEP12(2013)089]

EW vacuum
is the global minimum
in the effective potential

Although EW vacuum
isn’t the global minimum,
the lifetime is longer than
the age of universe

Neutrino oscillation and Our Purpose

Neutrino oscillation measurements imply that neutrinos have a tiny mass.

In SM, neutrinos are massless particles.
SM cannot explain massive neutrinos.

If SM is extended to explain a tiny neutrino mass,
we would like to investigate
how neutrino contributes to the lifetime of EW vacuum.

We introduce a heavy right-handed Majorana neutrino.
it leads to the “seesaw mechanism”.

$$\left(\overline{(\nu_L)^c} \quad \bar{\nu}_R\right)\begin{pmatrix}0 & m\\ m & M\end{pmatrix}\begin{pmatrix}\nu_L\\ (\nu_R)^c\end{pmatrix}$$

m : Dirac neutrino mass
M : Majorana neutrino mass

diagonalization of
ν mass matrix

$$\longrightarrow \quad (\nu \text{ mass eigen values}) = \frac{\mathrm{m}^2}{\mathrm{M}}, \mathrm{M}$$

Meta-Stability at tree level in Type-I seesaw model

The lifetime of EW vacuum is written as follow

$$\boxed{\frac{\tau}{T_U} = \left(\frac{R}{T_U}\right)^4 e^{S_b(\phi_b)} e^{\Delta S(\phi_b)}}$$

$$\frac{\tau}{T_U} = \underbrace{\left(\frac{R}{T_U}\right)^4 e^{S_b(\phi_b)}}_{\text{LO contribution}} \underbrace{e^{\Delta S(\phi_b)}}_{\text{NLO contribution}}$$

τ : the lifetime of EW vacuum
T_U : the age of universe
R : the bounce size

$$S_b(\phi_b) = \int d^4x \left[\frac{1}{2}(\partial_\mu \phi_b)^2 + V(\phi_b)\right] \qquad V(\phi_b) = \frac{\lambda}{4}\phi_b^4$$

bounce solution ϕ_b : $\phi_b(r) = \sqrt{\frac{8}{|\lambda(\mu)|}}\frac{R}{r^2+R^2}$

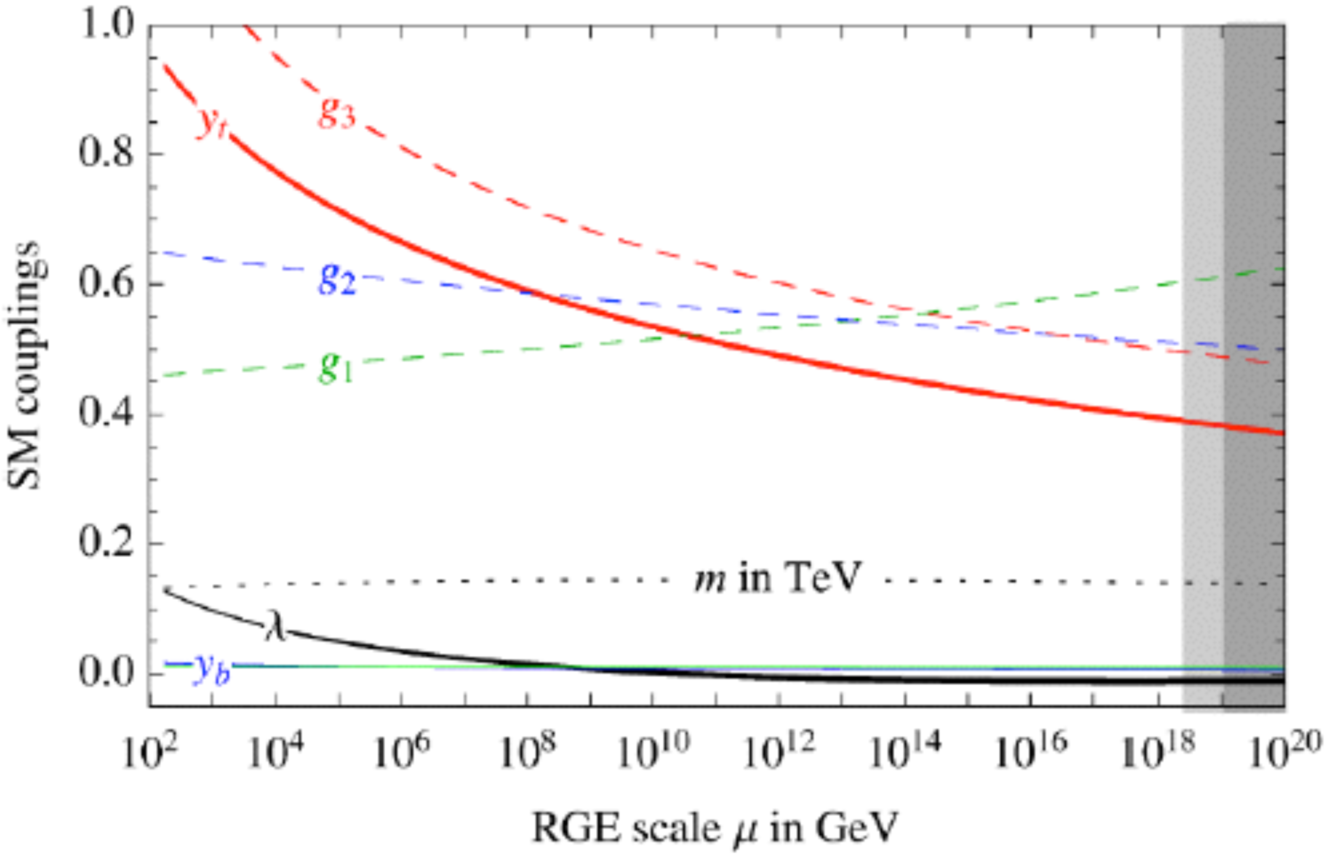
bounce solution satisfy
the equation of motion for Higgs field

$$\boxed{S_b(\phi_b) = \frac{8\pi^2}{3|\lambda(\mu)|}}$$

LO contribution to the lifetime of EW vacuum is mainly determined by the Higgs quartic coupling constant λ.

Magnitude of coupling constants

Dario Buttazzo et.al[JHEP12(2013)089]



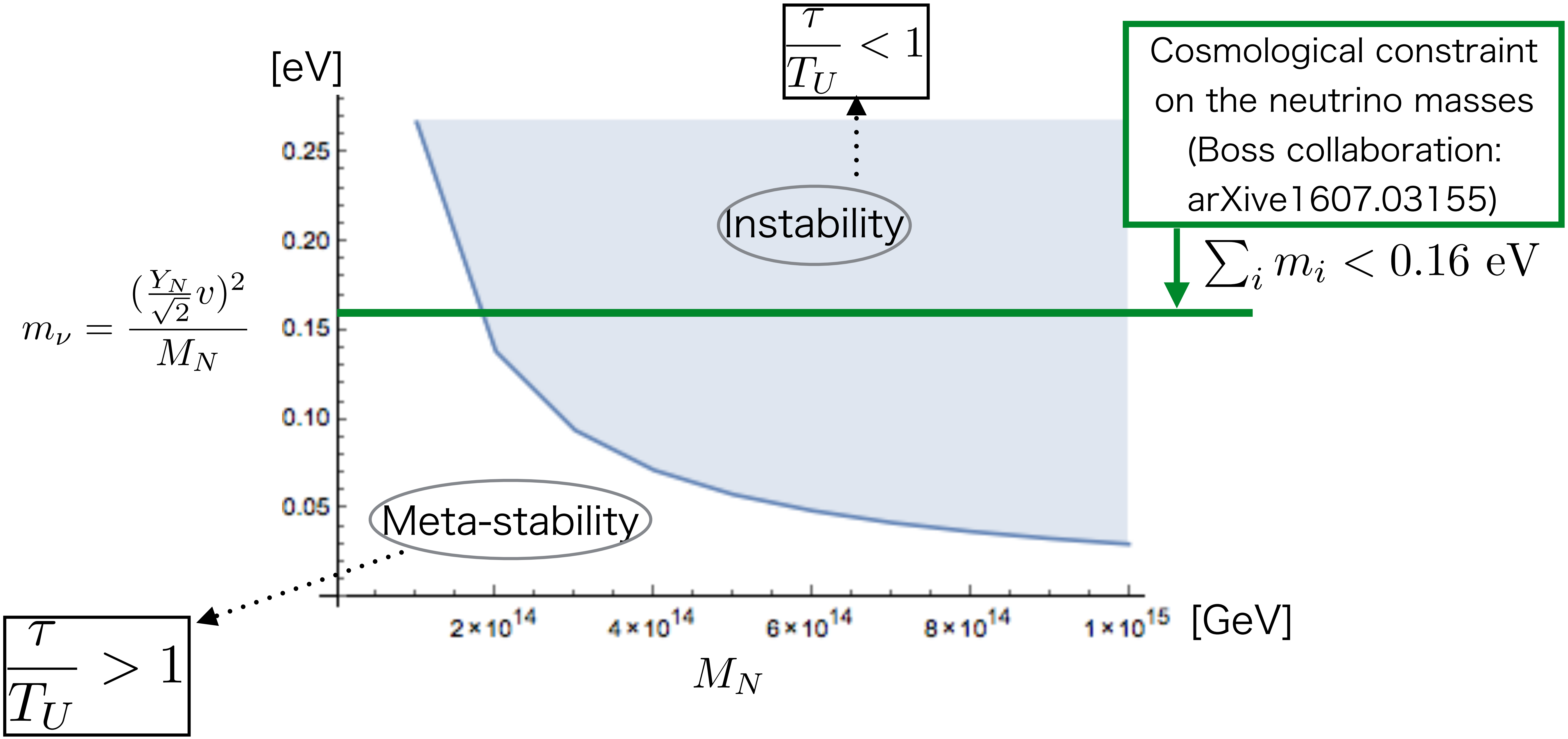
As shown from figure,
top Yukawa coupling y_t and
gauge couplings g_1, g_2 and g_3
are relevant to the running of λ

If a right-handed Majorana neutrino is heavy enough for neutrino Yukawa coupling constant Y_N to be comparable with y_t, g_1, g_2 and g_3,
the effect of a neutrino on the running of the coupling λ cannot be negligible.

We can expect that there can be the effect of a neutrino on the meta-stability.

Meta-Stability in LO

Requiring that the lifetime of EW vacuum is longer than the age of universe, we can constrain on the active neutrino mass.



$$\frac{\tau}{T_U} < 1$$

Cosmological constraint on the neutrino masses (Boss collaboration: arXive1607.03155)

$$\sum_i m_i < 0.16 \text{ eV}$$

$$m_\nu = \frac{(\frac{Y_N}{\sqrt{2}}v)^2}{M_N}$$

$$\frac{\tau}{T_U} > 1$$

Loop corrections to the lifetime of EW vacuum

- Higgs
- Top
- Neutrino

Calculation of the loop correction to the lifetime

Ishidori et.al, Nuclear Physics B 609 (2001) 387–409

$$\boxed{\Delta S = \frac{1}{2}\underline{ln\left(\frac{Det[-\partial^2+W(r)]}{Det[-\partial^2]}\right)} - \Delta S_{counter}}$$

$W(r)$: interaction term

$\downarrow$

It is usually difficult to calculate directly the eigenvalues of $-\partial^2+W(r)$ in order to evaluate $Det[-\partial^2+W(r)]$. However, we can evaluate the ratio $\frac{Det[-\partial^2+W(r)]}{Det[-\partial^2]}$ by Gelfand Yaglom theorem.

Roughly speaking, Gelfand Yaglom theorem says that the calculation of $Det[-\partial^2+W(r)]$ is replaced by solving the differential equation $[-\partial^2+W(r)]\psi=0$ with initial conditions.

We insert $\Delta S^{[2]} = \frac{1}{2}ln\left(\frac{Det[-\partial^2+W(r)]}{Det[-\partial^2]}\right)\Big|_{\mathcal{O}(W^2)}$ in order to regularize the correction to the action and renormalize the coupling constants with $\overline{MS}$

$$\Delta S = \underbrace{\left[\frac{1}{2}ln\left(\frac{Det[-\partial^2+W(r)]}{Det[-\partial^2]}\right) - \Delta S^{[2]}\right]}_{\text{numerical calculation (Gelfand Yaglom theorem)}} + \underbrace{\left[\Delta S^{[2]} - \Delta S_{counter}\right]_{\overline{MS}\text{ scheme}}}_{\text{analytical calculation (dimensional regularization)}}$$

Loop correction of Higgs to the lifetime

Ishidori et.al, Nuclear Physics B 609 (2001) 387–409

Gelfand Yaglom theorem

If one uses a Gelfand Yaglom theorem,
the eigenvalues problem is induced to solving the differential equation.

$$\frac{Det[-\partial^2 + W_H(r)]}{Det[-\partial^2]} = \prod_J \lim_{r\to\infty} \rho_J(r)^{(2J+1)^2}$$

$$\rho_j''(r) + \frac{4j+3}{r}\rho_j'(r) = W(r)\rho_j(r)$$

$$\rho_j(0) = 1,\ \rho_j'(0) = 0$$

$$\left[\frac{1}{2}ln\left(\frac{Det[-\partial^2 + W_H(r)]}{Det[-\partial^2]}\right) - \Delta S^{[2]}\right]_{Higgs} \approx 12.6$$

Dimensional regularization

$$[\Delta S^{[2]} - (\Delta S)_{\mathrm{pole}}]_{\mathrm{Higgs}} = -\frac{9\lambda^2}{64\pi^2}\int\frac{d^4q}{(2\pi)^4}[\widetilde{h^2}(q^2)]^2\left[2+\ln\frac{\mu^2}{q^2}\right] = -3L - \frac{5}{2}$$

$$L = \ln(R\mu e^{\gamma_E}/2)$$

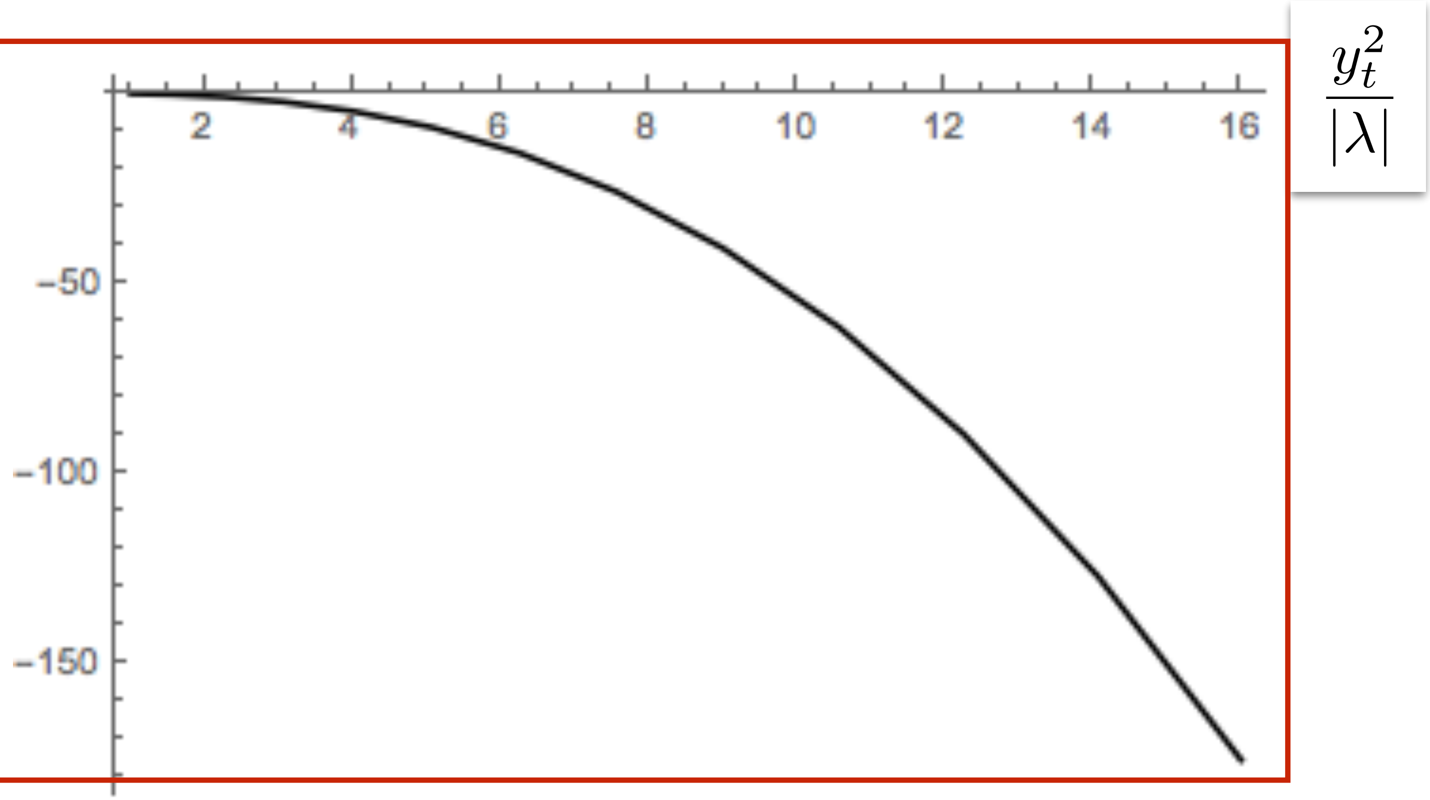
Loop correction of Top to the lifetime

$$\frac{Det[-\partial^2+W(r)]}{Det[-\partial^2]}=\prod_{J\geq\frac{3}{2}}\left[\lim_{r\to\infty}det\begin{bmatrix}\rho^1_{1J}(r) & \rho^2_{1J}(r)\\ \rho^1_{2J}(r) & \rho^2_{2J}(r)\end{bmatrix}^{2(J^2-\frac{1}{4})}\right]$$

$$\rho''_{1J}+2\frac{J}{r}\rho'_{1J}=\frac{g_t^2}{2}h^2\rho_{1J}+\frac{g_t}{\sqrt{2}}h'\rho_{2J}r,$$

$$\rho''_{2J}+2\frac{J+1}{r}\rho'_{2J}=\frac{g_t^2}{2}h^2\rho_{2J}+\frac{g_t}{\sqrt{2}}h'\rho_{1J}r^{-1}$$

$$\left[-\frac{3}{2}ln\left(\frac{Det[-\partial^2+W_t(r)]}{Det[-\partial^2]}\right)-\Delta S^{[2]}\right]_{top}$$



The correction of top to the action is negative.
So, this correction reduce the lifetime of EW vacuum.

$$[\Delta S^{[2]}-(\Delta S)_{\text{pole}}]_{\text{top}}=\frac{g_t^4}{6|\lambda|^2}(5+6L)+\frac{g_t^2}{6|\lambda|}(13+12L).\qquad L=\ln(R\mu e^{\gamma_E}/2)$$

Loop correction of neutrino to the lifetime

$$\mathcal{L}_\nu = -\overline{l_L}\tilde{H}Y_N\nu_R - \frac{M}{2}\overline{\nu_R}\nu_R^c + c.c.$$

$$\left[\frac{\mathrm{Det}[-\partial^2 + W_\nu(r)]}{\mathrm{Det}[-\partial^2]}\right]_{\mathrm{neutrino}} = \prod_{J\geq\frac{3}{2}}\left[\lim_{r\to\infty}\det\begin{pmatrix} \rho^1_{1J} & \rho^2_{1J} & \rho^3_{1J} & \rho^4_{1J} \\ \rho^1_{2J} & \rho^2_{2J} & \rho^3_{2J} & \rho^4_{2J} \\ \rho^1_{3J} & \rho^2_{3J} & \rho^3_{3J} & \rho^4_{3J} \\ \rho^1_{4J} & \rho^2_{4J} & \rho^3_{4J} & \rho^4_{4J} \end{pmatrix}\right]^{2\left(J^2-\frac{1}{4}\right)}$$

Similarly to Higgs and top cases,

We find the differential equations to evaluate the determinant.

$$\begin{pmatrix} -d_r + \frac{4}{r^2}J(J+1) & & \frac{y_\nu}{\sqrt{2}}M_N\phi_b & \frac{y_\nu}{\sqrt{2}}\phi_b' \\ & -d_r + \frac{4}{r^2}(J^2 - \frac{1}{4}) & \frac{y_\nu}{\sqrt{2}}\phi_b' & \frac{y_\nu}{\sqrt{2}}M_N\phi_b \\ \frac{y_\nu}{\sqrt{2}}M_N\phi_b & \frac{y_\nu}{\sqrt{2}}\phi_b' & -d_r + \frac{4}{r^2}J(J+1) + M_N^2 & \\ \frac{y_\nu}{\sqrt{2}}\phi_b' & \frac{y_\nu}{\sqrt{2}}M_N\phi_b & & -d_r + \frac{4}{r^2}(J^2 - \frac{1}{4}) + M_N^2 \end{pmatrix}\begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ \rho_4 \end{pmatrix} = 0$$

$$-d_r = -\frac{\partial^2}{\partial r^2} - \frac{3}{r}\frac{\partial}{\partial r} + \frac{1}{2}y_\nu^2\phi_b^2$$

If ϕ_b is replaced by $v_{EW}(\to \phi_b' = 0)$ and diagonalize this matrix, we can reproduce the seesaw mechanism. But in the lifetime calculation, we don't need to require the seesaw mechanism because ϕ_b can be so large value that Dirac mass $\frac{Y_N^2\phi_b^2}{2} \approx$ or $>$ Majorana mass M.

Summary

- SM guarantees the EW vacuum stability because of the long lifetime.

- If a right-handed Majorana neutrino mass is heavy,
neutrino Yukawa coupling can be comparable with relevant SM couplings.

- In that case, neutrino affects the lifetime of EW vacuum and
the active neutrino mass is constrained for the vacuum stability.

- If we estimate loop corrections from neutrino and gauge sectors,
we can discuss more accurately the effect of neutrino on the meta-stability.